

Uncertainty, Stochastics & Sensitivity Analysis

Nathaniel Osgood

MIT 15.879

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Types of Sensitivity Analyses

- Variables involved
 - One-way
 - Multi-way
- Type of component being varied
 - Parameter sensitivity analysis: Parameter values
 - Structural sensitivity analysis: Examine effects of model *structure* on results
- Type of variation
 - Single alternative values
 - Monte Carlo analyses: Draws from probability distributions (many types of variations)
- Frequency of variation
 - Static (parameter retains value all through simulation)
 - Ongoing change: Stochastic process
 - Accomplished via Monte-Carlo analyses
 - Key for DES & ABM

Model Uncertainty

- Here, we are frequently examining the impact of changing
 - Our assumptions about “how the system works”
 - Our decision of how to abstract the system behaviour
- Structural sensitivity analyses
 - Vary structure of model & see impact on
 - Results
 - Tradeoffs between choices
 - Frequently recalibrate the model in this process
- Here, we are considering uncertainty about how the current state is mapped to the next state

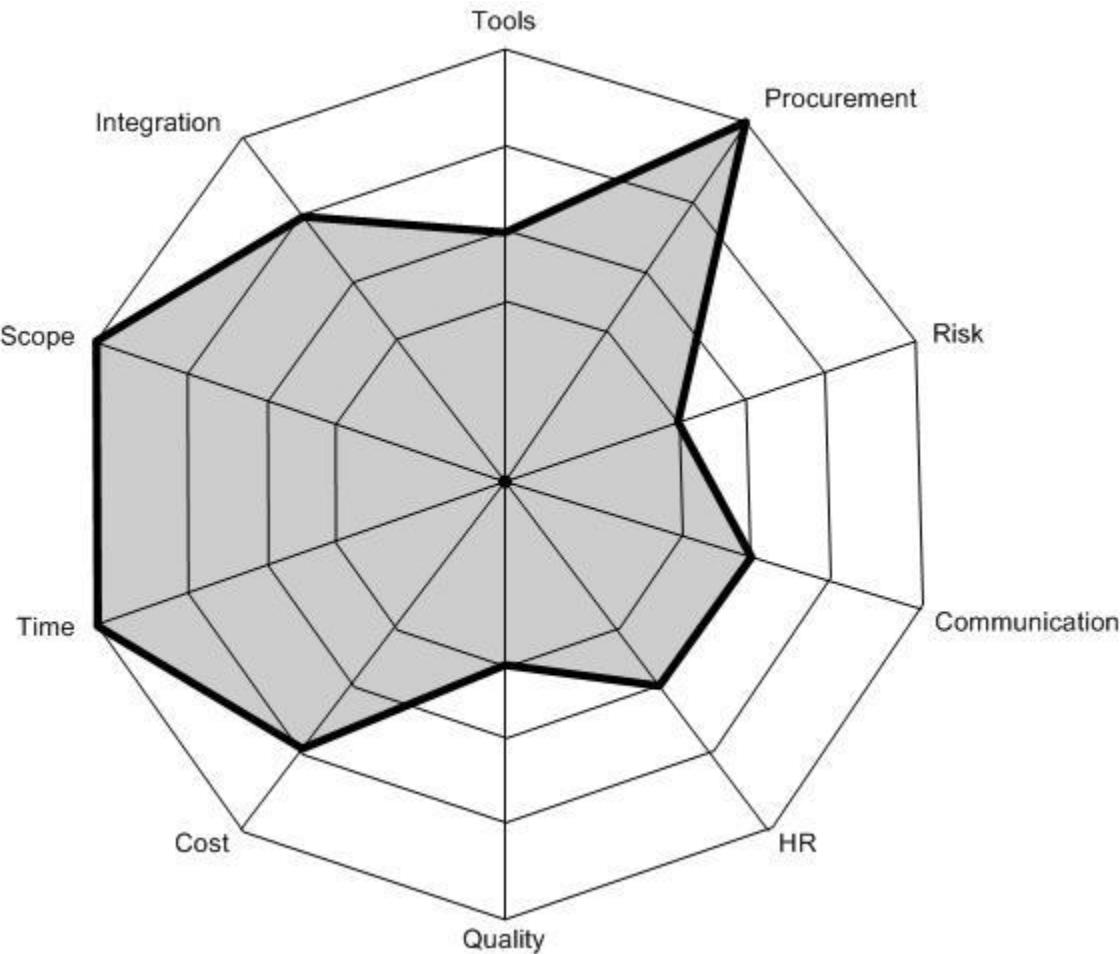
Predictor-Corrector Methods: Dealing with an Incomplete Model

- Some approaches (e.g. Kalman filter, Particle Filter) are motivated by awareness that models are incomplete
- Such approaches try to adjust model state estimates on an ongoing basis,
 - Given uncertainty about model predictions
 - New observations
- Assumption here is that the error in the model is defined by some probability distribution

Static Uncertainty Sensitivity Analyses

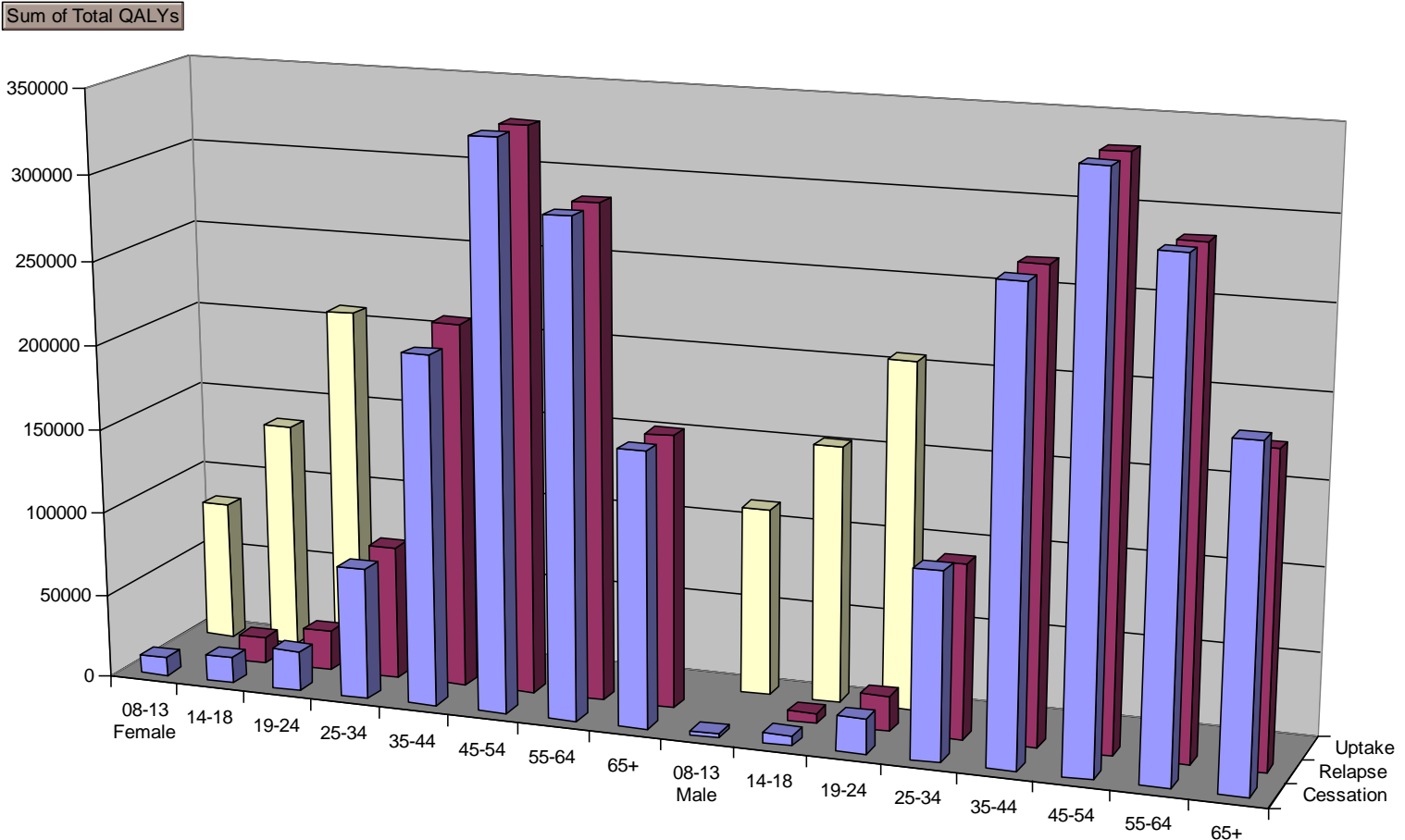
- In variation, one can seek to investigate different
 - Assumptions
 - Policies
- Same relative or absolute uncertainty in different parameters may have hugely different effect on outcomes or decisions
- Help identify parameters/initial states that strongly affect
 - Key model results
 - Choice between policies
- We place more emphasis in parameter estimation & interventions into parameters exhibiting high sensitivity

Spider Diagram



- Each axis represents a % change in a particular parameter
 - This proportional change is identical for the different parameters
- The distance assumed by the curve along that axis represents the magnitude of response to that change
 - Note that these sensitivities will depend on the state of system!

Systematic Examination of Policies



Add New “Parameters Variation” Experiment

New Experiment

Experiment
Select an experiment type, specify a name and choose a root (top-level) active object.

Name:

Main Active Object Class (root):

Experiment Type:

- Simulation
- Optimization
- Parameters Variation
- Compare Runs (available in Professional edition)
- Monte Carlo (available in Professional edition)
- Sensitivity Analysis (available in Professional edition)
- Calibration (available in Professional edition)
- Custom (available in Professional edition)

Performs multiple model runs varying one or more parameters, optionally using replications.
You can later on add arbitrary UI to this experiment.

Copy model time settings from :

< Back Next > Finish Cancel

Setting Ranges for Parameter Variation Can Handle 1-Way or (Orthogonal) Multi-Way

The screenshot displays the AnyLogic Advanced software interface. The main window shows a grid for modeling. The left sidebar contains a project tree with folders like 'Presentation', 'Simulation: Main', and 'Parameters'. The bottom panel is titled 'ParametersVariation - Parameter Variation Experiment' and contains the following settings:

- Name:** ParametersVariation
- Main active object class (root):** Main
- Random number generation:** Random seed (unique simulation runs), Fixed seed (reproducible simulation runs) with Seed Value: 1
- Parameters:** Varied in range, Freeform, Number of runs: 10

Parameter	Type	Value		
		Min	Max	Step
AverageI...uration	range	0	10	1
ContactRate	fixed	1.0		
Infection...bability	range	0.8		
AreaSide	fixed	100		
TotalPopulation	fixed	10000		

The right sidebar shows a 'Model' palette with various components like Parameter, Flow Aux Variable, Stock Variable, Event, Dynamic Event, Plain Variable, Collection Variable, Function, Table Function, Port, Connector, Entry Point, State, Transition, Initial State Pointer, Branch, History State, Final State, and Environment. At the bottom right, there are buttons for Action, Analysis, Presentation, Connectivity, Enterprise Library, and More Libraries...

Sensitivity Exploration in AnyLogic

Performing 1 Way Sensitivity (for now...)

The screenshot displays the AnyLogic Advanced software interface. The main window shows a Monte Carlo simulation titled "Agent Based SIR Model - Monte Carlo Simulation". The simulation results are visualized as a 2D histogram with a grid. The y-axis ranges from 1,500 to 4,000. A button labeled "Run 100 replicat..." is visible. The software interface includes a menu bar (File, Edit, View, Model, Window, Help), a toolbar, and a project tree on the left. The bottom panel shows the configuration for the "MonteCarlo2DHistogram - Parameter Variation Experiment".

MonteCarlo2DHistogram - Parameter Variation Experiment

General: Name: MonteCarlo2DHistogram Main active object class (root): Main

Advanced: Random number generation: Random seed (unique simulation runs) Fixed seed (reproducible simulation runs) Seed Value: 1

Parameters: Varied in range Freeform Number of runs: 100

Parameter	Type	Value		
		Min	Max	Step
AverageI...uration	range	0	10	1
ContactRate	fixed	1.0		
Infection...bability	fixed	0.8		
AreaSide	fixed	100		
TotalPopulation	fixed	10000		

Monte Carlo Analyses in AnyLogic: Specifying Distributions for Parameters

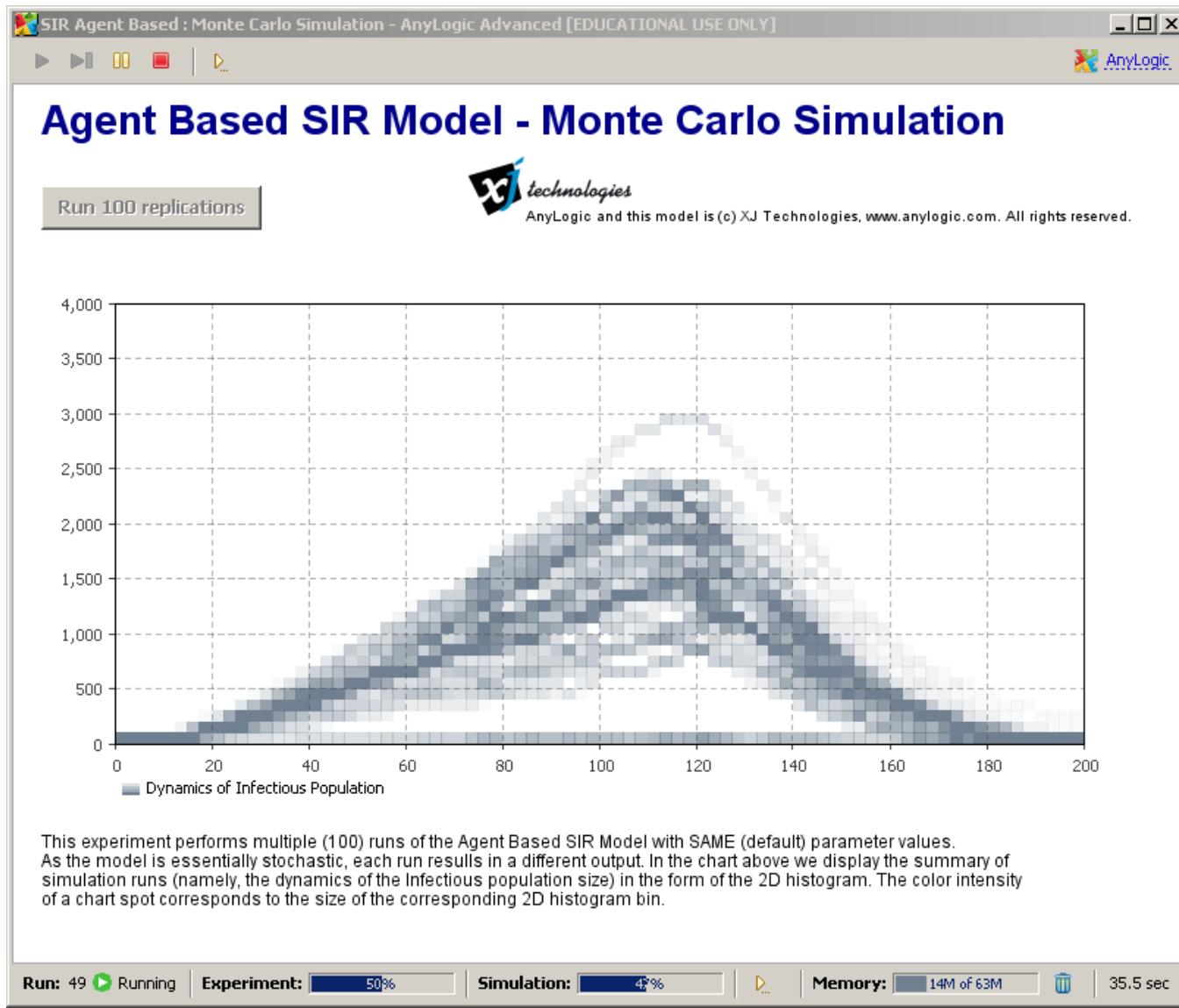
The screenshot shows the AnyLogic Advanced [EDUCATIONAL USE ONLY] interface. The main window displays a simulation titled "Agent Based SIR Model - Monte Carlo Simulation". A button labeled "Run 100 replicat..." is visible. The simulation area shows a 2D histogram of "infectious2D" values, with the y-axis ranging from 2,000 to 4,000. The histogram shows a distribution of values across the simulation runs.

The "MonteCarlo2DHistogram - Parameter Variation Experiment" properties window is open, showing the following settings:

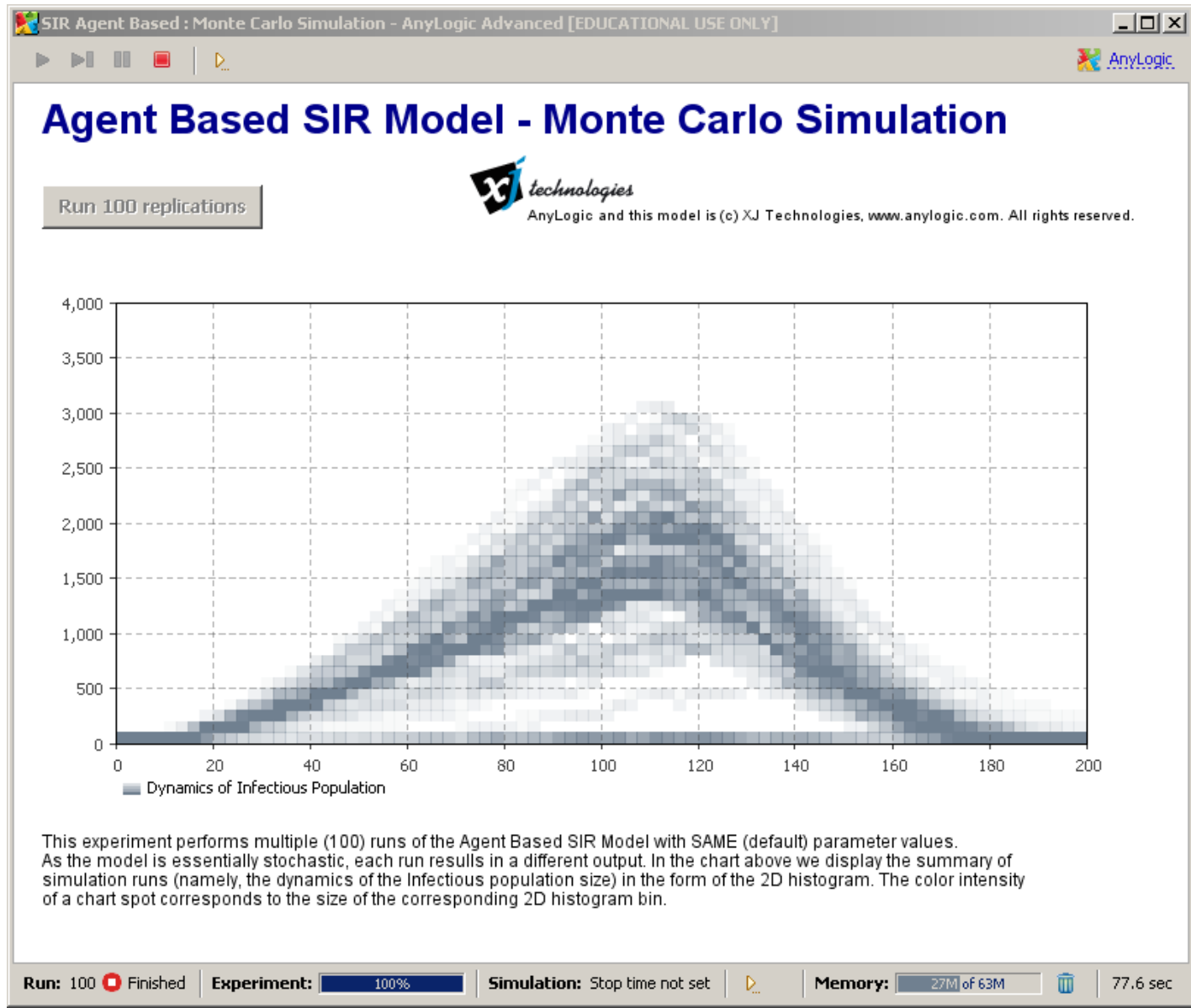
- Name: MonteCarlo2DHistogram
- Main active object class (root): Main
- Ignore:
- Create Default U:
- Random number generation:
 - Random seed (unique simulation runs)
 - Fixed seed (reproducible simulation runs) Seed Value: 1
- Parameters: Varied in range Freeform Number of runs: 100

Parameter	Expression
Average...ration*	max(0,normal(5,15))
ContactRate	1.0
Infection...bability	0.8
AreaSide	100
TotalPopulation	10000

Monte Carlo Output After Some Runs



Monte Carlo Output After All Runs



Sensitivity in Initial States

- Frequently we don't know the exact state of the system at a certain point in time
- A very useful type of sensitivity analysis is to vary the initial model state
- In Aggregate models, this can be accomplished by
 - Varying the number of people in the stock via a parameter to adjust
- In an agent-based model, state has far larger dimensionality
 - Can modify different numbers of people with characteristic, location of people with characteristic, etc.

Imposing a Probability Distribution

Monte Carlo Analysis

- We feed in probability distributions to reflect our uncertainty about one or more parameters
- The model is run many, many times (realizations)
 - For each realization, the model uses a different draw from those probability distribution
- What emerges is resulting probability distribution for model outputs

Multi-Way Sensitivity Analyses

- When examining the results of changing multiple variables, need to consider how multiple variables vary together
- If this covariation reflects dependence on some underlying factor, may be able to simulate uncertainty in underlying factor